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FACULTY WORKING
PAPER NO. 907

Incentives for Competitive Responses in Large Economies: A Reformulation

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October 1982

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Abstract

This paper extends some earlier results of Roberts and Postlewaite on truthful revelation of preferences for economies with a finite number of types and corrects their conclusions concerning infinite economies.

INCENTIVES FOR COMPETITIVE RESPONSES IN LARGE ECONOMIES: A REFORMULATION

I. In 1976 D. J. Roberts and A. Postlewaite (hereafter RP) considered the following question: Can an agent gain by providing the Walrasian auctioneer with a "demand" curve that is not generated by his true utility function? While it is well-known that such falsification is possible and profitable in economies with few agents RP asked whether the gains from such falsification went to zero as the number of agents increased without bound. In the case of replicated economies they proved that it is indeed the case that the gains from falsification went to zero asymptotically. By an example, they next showed that there were families of economies in which an agent could always gain a fixed amount (in utility terms, for some fixed utility function) by falsifying his preferences. Finally, they provided a sufficient condition for families of economies to provide asympototically zero gains from falsification for any given agent.[1]

The last part of the RP paper is, unfortunately, open to question. To prove their major theorem, (Theorem 2) they consider economies with a continuum of agents, in which all individuals are of measure zero, hence negligile. Since our aim is to consider manipulation by individuals it seems odd to turn to a formalization in which individuals must, of necessity, be negligible. We will provide a different proof of the RP result for replicated economies, which will permit us to both generalize the result to any type economy as well as lead us to a general condition for manipulation to be locally profitable. It will be seen that the possibility of locally profitable manipulation depends upon the slope of the excess demand function rather than upon the continuity of the equilibrium set correspondence. (We have been unable to find global conditions for this problem, however.) Readers familiar with the Roberts-Postlewaite paper will have no difficulty discovering our considerable debt to them.

By formulating the question in a Nonstandard-Analytic framework it can be shown that even in economies with an infinity of agents, (indeed, an infinity equal to any preassigned cardinal) individuals can still possess finite market power. This result is of course counter to our intuition, which suggests that, with many agents in an economy, each individual should have negligible influence. Intuitions however are not proofs and need to be tested against rigorous analysis. What turns out to be important is the concept of similarity: If there are many agents "similar" to the agent in question then the agent will have no manipulative power; if, however, the agent in question is of an isolated type, an outlier, then his manipulative capacities need not vanish in large economies.

Before proceeding, it is important to make a point that is theoretically trivial but of significance in interpreting results on manipulability. Suppose an economy has multiple competitive equilibria, denoted, for the sake of simplicity by $p_1, \ldots, p_k$. In general, utility of the manipulative agent, denoted a, will be higher at some equilibria than at others. Let us relabel prices so that p_1 gives a most utility and p_k least. If a now gives out a false demand function at prices $p_2, \ldots, p_k$ but his true demand function at price p_1 then p_1 will appear to be the unique equilibrium price vector for the economy.[2] Even though a has given out his true competitive response at price p_1, it is clear that a has manipulated the economy in order to provide an outcome favorable to himself. Since this is always possible in economies with multiple equilibria and since uniqueness is quite rare, it follows that manipulation will, in general, be feasible and profitable.

The description of the model is very closely adapted to that of RP. Without loss of generality we make an observation that will considerably simplify the analysis and also relax the boundary conditions imposed by RP. The focus throughout is on the manipulative capabilities of agent a. If all prices are positive, the possible responses of a, which will be limited by his budget, will of course be bounded. If, on the other hand, $p_n = 0$ then either a has a satiation level z_n for this commodity, in which case his response is again bounded,[3] or his response will be unbounded, in which case no competitive equilibrium can exist and we need not consider such prices.

We consider pure exchange situations with a fixed number, N, of commodities. In such situations, an economic agent is characterized by his needs, his tastes, and his ownership of resources. These characteristics are specified mathematically by a (non-empty) consumption set $X \subset R^N_+$, a preference relation $\gtrsim$ on X, and an endowment vector $w \in R^N_+$. We will write $\gtrsim(a)$, X(a), and w(a), respectively, for the preferences, consumption set, and endowment of an agent a.

Let P denote the standard unit simplex in R^N_+. The competitive, price-taking response gives, for each possible agent and each price, the set of net trades which are preference maximizing for the agent given those prices. Formally, given $a \in A$ and $p \in P$,

$C(a,p) \equiv \{z \in R^N \mid z - x - w(a)$ and x is $\succsim(a)$-maximal on $X(a)$ subject to $px \leqq pw(a)\}$. (Although we do not assume $C(a,p) \neq \emptyset$, conditions sufficient for non-emptiness of $C(a,p)$ are well known. Note that $C(a,p)$ is closed for all $a \in A$ and $p \in P$.)

To allow for deviations from passive price-taking behavior, we permit agents to select responses to prices other than those specified by the competitive response. Specifically, let $L(a)$ denote the collection of all correspondences S from P into R^N with the properties that, for all $p \in P$, $S(p) + \{w(a)\} \subset X(a)$ and, if $z \in S(p)$, then $pz = 0$. We think of these correspondences as possible strategies an agent can employ in departing from competitive rules; agents may adopt any form of non-competitive behavior which can be described by a correspondence from prices to net trades, so long as they do not violate their budget constraints or the constraints imposed by their endowments.

A finite exchange economy is a finite collection of agents and an assignment of a response correspondence to each. Thus, we can represent E as a mapping that assigns to each i in some finite index set I a point (a_i, S_i), where $a_i \in A$ and $S_i \in L(a_1)$. Given an economy E with response correspondences $S_1, \ldots, S_M$, we say that $p \in P$ is a market-clearing price if there exist $z_m \in S_m(p)$ such that $0 = \Sigma_{m=1}^{M} z_m$. Denote by $Q(E)$ the set of market-clearing prices for E. Note that $Q(E)$ is determined by the response correspondences S_m. If for each agent a_m the response S_m is his competitive response, then $Q(E)$ is just the usual set of competitive equilibrium prices.

Given the response correspondences of the other agents, an agent a_i can, by adopting a response correspondence S^* other than that specified initially for him, effectively create a new "apparent economy" E^* in which (a_i, S_i) has been replaced by (a_i, S^*). Generally, $Q(E^*)$ will differ from $Q(E)$. We say that, given an economy E with response functions $S_1, \ldots, S_M$ and given an agent a_i in E, a price $\bar{p}$ is attainable for a_i in E if there exists $S^* \in L(a_i)$ such that $0 \in \Sigma_{m \neq i} S_m(\bar{p}) + S^*(\bar{p})$, that is, if $\bar{p} \in Q(E^*)$ for some "apparent economy" which a_i can effect.

We define the competitive response as <u>individually incentive compatible</u> for an agent a in E if, for any consumption vector x attainable by a in E, there exists a competitive consumption y for a such that $y \geq (a)x$.

II. We begin by considering Replicated Economies. Let there be M types, $m = 1, \ldots, M$. We focus upon a particular agent a, of type M. True excess demand responses will be denoted by z_i. The false responses of a will be denoted by y.

Theorem 1: If a's indirect utility function v(p) is continuous, then his gain from misrepresentation becomes asymptotically zero in a sequence of replicated economies.

Proof: Two proofs are given, the first one being a simplification of RP. Let y_k denote the false response in the kth replication and p_k' the resulting equilibrium.

$$kz_1 + \ldots + kz_{m-1} + (k - 1)z_m + y_k' = 0$$

or
$$kz_1 + \ldots + kz_{m-1} + kz_m + y_k' - z_m = 0$$

or
$$z_1 + \ldots + z_m + \frac{y_k' - z_m}{k} = 0.$$

$\frac{y_k' - z_m}{k} \to 0$ as $k \to \infty$ because both y_k' and z_m are bounded.

Hence $\Sigma z_i(p_k') \to 0$ as $k \to \infty$ or p_k' tends to an equilibrium price vector p^*. The continuity of v(p) now establishes the theorem.

Q.E.D.

For the second proof, let $p' \neq p^*$ and $\{z_i'\}$,y corresponds to p'.

$$z_1' + \ldots + z_m' = \varepsilon = 0 \text{ since } p' \text{ is not an equilibrium price.}$$

In the k-fold replication,

$$kz_1' + \ldots + kz_{m-1}' + kz_M' = k\varepsilon.$$

or $$kz_1' + \ldots + kz_{m-1}' + (k-1)z_M' + y = k\varepsilon - z_M' + y(*).$$

If $y(p')$ is a manipulative response,

$$kz_1' + \ldots + kz_{m-1}' + (k-1)z_M' + y = 0.$$

So $k\varepsilon - z_M' + y = 0$, but $||k\varepsilon|| \to \infty$ as $k \to \infty$ while z_M' and y are bounded. Hence any $p' \neq p^*$ is infeasible for large k, with is another way of proving Theorem 1.

Q.E.D.

Before trying to generalize this result let us extract a property which is useful later in characterizing local manipulability. If ED indicates the aggregate excess demand and $p' = p^* + \delta$, then (*) implies

$\frac{||ED(p^* + \delta) - ED(p^*)||}{||\delta||} = k\frac{||\varepsilon||}{||\delta||} \to \infty$ as $k \to \infty$. So ED has infinite slope as $k \to \infty$.

In order to generalize Theorem 1 it is worth considering the following equation, (*), again,

$$k\varepsilon - z_M' + y = 0,$$

which can be rewritten as

$$(\text{Replicated response} = k\varepsilon) + (\text{Non-replicated} = y - z_M') = 0.$$

If the replicated response tends to infinity as the number of traders increase, while the non-replicated response stays bounded, then the argument of Theorem 1 can be repeated. Using Nonstandard analysis, all this can be proved and we state it informally as

Theorem 2: The number of types of agents is finite while the number of agents is infinite. Let $\{1, \ldots, M\}$ denote those types which are represented in non-negligible proportions in the infinite economy and let Π_M denote the equilibrium price vectors for the finite economy consisting only of types $\{1, \ldots, M\}$. Then any price vector achievable through manipulation in the infinite economy must lie infinitesimally close to Π_M.

Proof: See Appendix.

If agents who are similar demand similar commodity bundles, a sufficient condition for this being strict convexity of preferences, then Theorem 2 can be extended to economies whose types are chosen from a compact set. This result is formally stated in the Appendix as Theorem 3 and proved therein.

Theorems 2 and 3 leave open the possibility that a trader who is an outlier, i.e., one who does not have many traders similar to himself, can manipulate an economy. Indeed, this is borne out by the RP example, in which the agent who successfully manipulates is the only one with his characteristics regardless of how large the economy becomes. The importance of considering economies in which the number of types is limited has been previously emphasized in treating the core equivalence theorem and is now reached again in our attempt to formalize the intuitive idea that individuals become insignificant in large economies.

We now turn to discussing the difference between our approach and that of RP.

III. Let $\overline{ED}$ denote the aggregate excess demand of all traders except for a. For manipulation to be feasible in an economy of <u>any</u> size,

$$\overline{ED}(p) + y'(p) = 0.$$

Note that this condition is independent of the size of the economy and there is no presumption that such an equation cannot be satisfied in economies with many agents.

<u>Theorem 4</u>: If $\overline{ED}(p^*)$ is continuous and has finite rate of change (= derivative, when a derivative exists), and if $z_a(p^*)$ is an interior point of 2's consumption set then manipulation to achieve p', with $-\delta_2 \leq p^* - p' \leq \delta$, is feasible. If $z_a(p^*)$ is a boundary point, then only one-sided manipulation may be possible.

<u>Proof</u>: $\overline{ED}(p)$ is a continuous function of p. At p^*, $\overline{ED}(p^*) + z(p^*) = 0$, or $\overline{ED}(p^*) = -z(p^*)$. As p changes slightly from p^* the choices for a manipulative response y(p), which consists of all vectors on the budget line at prices p, move continuously. To put it differently, for $|p - p^*| < \delta$, we can choose y(p) such that $|y(p) - z(p^*)| < \varepsilon$. Hence, there exists a δ-neighborhood of p^* such that $\overline{ED}(p) + y(p) = 0$.

Q.E.D.

A diagram will help clarify the issue between RP and ourselves. In the case of two goods we can use Walras Law and consider only one good, with price P_1, as relevant to equilibrium. The line y(a) indicates the upper limit to $y_1(a)$ that can be purchased by a for each p_1. Obviously, values below this line are also feasible for a, so the correspondence of feasible false vectors consists of everything below the y(a) curve.

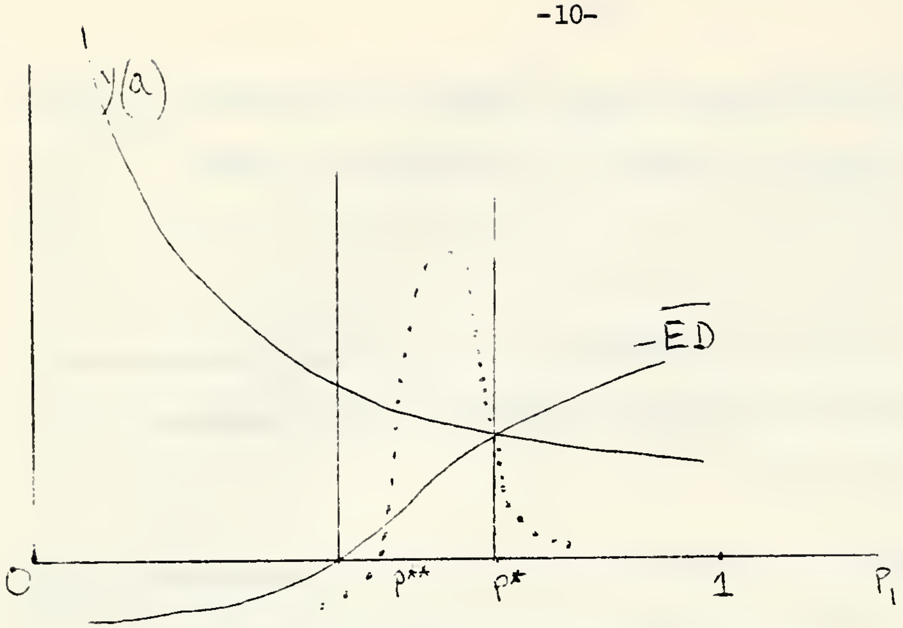

Figure 1

If $-\overline{ED}$ has the shape shown above then all the prices between the two parallel vertical lines can be achieved by manipulation. The proximate choice for a is a price vector p'.

<u>A1</u> Suppose as $k \to \infty$ the only feasible p' are such that $|p' - p^*| \to 0$.

Then it is obvious that the continuity of V at p* will ensure limiting incentive compatibility. A necessary condition for <u>A1</u> to hold is that $-\overline{ED}(p^*)$ has infinite slope at p*. It is not however sufficient, as shown by the dotted line in Figure 1, which has local incentive compatibility at p* but which nonetheless can be manipulated to get the price vector p**.

Note however that the continuity of the equilibrium set correspondence is <u>not</u> involved contrary to RP. By the recent results of Sonnenschein, McFadden et al we know that it is always possible to create an economy with a specified excess demand function. By using these results we can have $\overline{ED}$ as the <u>same</u> function for all k (and a constant correspondence is of course continuous). However, the diagram clearly shows that the same amount of manipulation remains possible in economies of arbitrary size. This point had been clearly demonstrated by the RP example, but they unfortunately blunt its impact by linking the example with the continuity of the equilibrium set correspondence.

The intuition behind the RP result may be stated as follows:

1. ξ is an economy in which there is zero gain from misrepresentation.
2. $\xi_k \to \xi$ and the equilibrium set correspondence is continuous.
3. Then the gain from misrepresentation must tend to zero as $k \to \infty$.

The questionable step here is no. 1. RP focus upon economies with a continuum of agents. It is true that there is no incentive to misrepresent in continuum economies according to the definition given. However, and this is crucial, in the definition of equilibrium in Aumann-Hildebrand economies, a set of excess demands is in equilibrium if its <u>average sum</u> is zero. The earlier definition however required the <u>exact sum</u> to be zero and in large economies the difference between $\frac{1}{k}\Sigma z_i \to 0$ as $k \to \infty$ and $\Sigma z_i = 0$ for all k is very great. Unfortunately, it is not possible to evade average sums using measure theory and it is for this reason that Nonstandard Analysis is a better tool for this problem.

IV. It has been argued in this paper that large economies are much more open to manipulation than had previously been thought possible. If we argue consistently that an equilibrium is defined by the aggregate excess demand (and not the average aggregate excess demand) being equal to zero, then an agent can always manipulate to his advantage if there are multiple equilibria and it is possible for him to manipulate whenever there are not "many" agents similar to himself. The weak point of the methodology pursued above, which is identical to that assumed by Roberts and Postlewaite in the first half of their paper, is its assumption that an individual can have access to all the information needed for successful manipulation. (This weakness is explicitly recognized by RP in footnote 4.) Indeed, in conclusion, it may not be amiss to state a result that has a very different flavor. If an economy is enlarged by drawing at random from the space of agents characteristics then the excess demand functions will all be identically distributed random variables. The aggregate excess demand $\overline{ED}(p)$ will, by the Central Limit Theorem, be normally distributed. If the manipulative agent chooses a false response $y_a(p)$ then the probability that $y_a(p) = -\overline{ED}(p)$ must be zero since the probability of a given point is zero for any nonatomic distribution. This shows that unless the agent has access to very extensive information the probability of successful manipulation is indeed very small.

NOTES

[1]In subsequent papers Roberts (2) considered the incentives problem for an economy with public goods while Hammond (3) considered the same problem for both private and public goods economies within a measure-theoretic framework. We avoid the problem of public goods in this paper.

[2]E.g., agent 2 could use the response $Y(p) = D(p) + \delta(p)$ where $D(p)$ is his true response and $\delta(p) = ||p - p_1||$.

[3]A direct argument establishes that an agent can never gain by pretending to have an infinite demand for a good which he can be satiated with.

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We are very grateful to Professor D. J. Roberts for the careful reading and constructive criticisms he provided on an earlier version of this paper.

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APPENDIX

The model is formulated as in the text of the paper except that we now have w agents, with $w \varepsilon {}^*N - N$. (For an introduction to Non-standard Analysis see Robinson (5).) The space of agents characteristics is topologized as in Hildenbrand (4). The infinite economy is denoted ξ^w and the number of agents of type i is denoted N_i.

<u>Theorem 2</u>: If $M = \{i|\frac{N_i}{w} > 0\}$ is finite and Π_M is the set of equilibrium price vectors for the finite economy consisting of these M types. If p' is a price vector achieved by manipulation in ξ^w then $p' \simeq \Pi_M$.

<u>Proof</u>: Since p' is achieved by manipulation $z_1(p') \ldots + z_{w-1}(p') + y(p') = 0$. Collecting together terms for agents in M,

$$k(z_1(p') \ldots + z_M(p')) + \sum_{i=M+1} \ell_i z_i(p') + y(p') = 0$$

where k $\;{}^*N - N$ and ℓ_i, z_i correspond to those numbers and excess demands of those types not in M. Since $\Sigma\, \ell_i z_i(p') + y(p')$ is bounded $k(z_1(p') + \ldots + z_M(p'))$ is bounded. As k $\;{}^*N - N$, $z_1(p') + \ldots + z_M(p') \simeq 0$. Hence p' is infinitesimally close to being an equilibrium price vector.

Q.E.D.

In proving Theorem 3 we shall make use of the following well-known fact about compact sets S. For all $\varepsilon > 0$, there exists a finite set M, such that only finite set of points do not lie within an ε-neighborhood of some member of M. The finite set M will be called the ε-approximation of S.

Theorem 3 : If agents characteristics in ξ^w are chosen from a compact subset S of the space of strictly convex characteristics and M denotes the ε-approximation of S, for "small" $\varepsilon > 0$, then any price vector p' achievable through manipulation is infinitesimally close to being an equilibrium price vector for agents.

Proof : From Hildenbrand (4) we know that agents whose characteristics are close will chose commodity bundles that are close. Hence

$$z_1 + \ldots + z_{w-1} + y = 0$$

can be written as

$$(z_1' + \frac{\varepsilon_1}{2}) + (z_1' + \frac{\varepsilon_1}{2^2}) + \ldots + (z_1' + \frac{\varepsilon_1}{2^n}) + \ldots$$
$$(z_M' + \frac{\varepsilon_M}{2} + \ldots + z_M' + \frac{\varepsilon_M}{2^n}) + z_{M+1} + \ldots z_{w-1} + y = 0$$

where z_i' are the responses of the limit point types. This can be rewritten as

$$\Sigma k_i z_i + \delta + \sum_{M+1} \ell z_i + y = 0$$

where $\frac{k_i}{w} \geq 0$.

The argument of Theorem 2 now applies as before.

Q.E.D.